Ambiguities in the HBT approach to determinations of interaction regions

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Introduction

M.A. Lisa et al., *Ann. Rev. Nucl. Part. Sci.* **55**(2005)357, K. Zalewski, *Acta Phys. Pol.* **B39**(2008)181

Homogeneity regions: profiles $p(\mathbf{x}|\mathbf{K})$

Assumption I: $$p(\mathbf{X},\mathbf{K}) = \int dt\, S(\mathbf{X},t,K)$$

$$p(\mathbf{X}|\mathbf{K}) \Leftrightarrow \left\langle \prod_{i=x,y,z} (x_i)^{n_i} \right\rangle \Leftrightarrow \left\langle \prod_{i=x,y,z} (x_i - \langle x_i \rangle)^{n_i} \right\rangle \equiv \mu_{n_x,n_y,n_z}$$

Cumulants $\mathcal{K}(r_x, r_y, r_z)$. Order $r = r_x + r_y + r_z$.
In one dimension the first four cumulants are

$$\langle x \rangle, \qquad \mu_2, \qquad \mu_3, \qquad \mu_4 - 3\mu_2^2.$$

Model

J.Karczmarczuk, Nucl. Phys.B78(1974)370, A. Bialas and K. Zalewski, Phys. Rev. D72(2005)036009, K. Zalewski, Phys. Rev. D74(2006)114022

Assumption II: $$P(\mathbf{p}_1, \ldots, \mathbf{p}_n) = C_n \sum_P \prod_{j=1}^{n} \rho(\mathbf{p}_j; \mathbf{p}_{Pj}).$$

E.g. $$P(\mathbf{p}_1, \mathbf{p}_2) = \rho(\mathbf{p}_1; \mathbf{p}_1)\rho(\mathbf{p}_2; \mathbf{p}_2) + |\rho(\mathbf{p}_1; \mathbf{p}_2)|^2.$$

All these distributions are invariant with respect to

$$\rho(\mathbf{p}_1; \mathbf{p}_2) \rightarrow \rho'(\mathbf{p}_1; \mathbf{p}_2) = \rho(\mathbf{p}_2; \mathbf{p}_1), \qquad \text{and/or}$$
$$\rho(\mathbf{p}_1; \mathbf{p}_2) \rightarrow \rho'(\mathbf{p}_1; \mathbf{p}_2) = e^{if(\mathbf{p}_1)}\rho(\mathbf{p}_1; \mathbf{p}_2)e^{-if(\mathbf{p}_2)}.$$

No other invariance.

Cumulants

Define: $\mathbf{K} = \frac{1}{2}(\mathbf{p}_1 + \mathbf{p}_2); \qquad \mathbf{q} = \mathbf{p}_1 - \mathbf{p}_2.$

$$\rho(\mathbf{K}, \mathbf{q}) = \rho(\mathbf{K}, \mathbf{0}) \int d^3X \; p(\mathbf{X}|\mathbf{K}) e^{-i\mathbf{q}\mathbf{X}}$$

At given $\mathbf{K}$, function $\frac{\rho(\mathbf{K},\mathbf{q})}{\rho(\mathbf{K},\mathbf{0})}$ is the characteristic function of the probability distribution $p(\mathbf{X}|\mathbf{K})$. Therefore, the cumulants of the probability distributions $p(\mathbf{X}|\mathbf{K})$ can be obtained from the expansion

$$\log\left(\frac{\rho(\mathbf{K},\mathbf{q})}{\rho(\mathbf{K},\mathbf{0})}\right) = \sum_{r_x, r_y, r_z} \frac{q_x^{r_x} q_y^{r_y} q_z^{r_z}}{r_x! r_y! r_z!} \mathcal{K}(r_x, r_y, r_z).$$

Result

K. Zalewski, *Phys. Rev..* **D77**(2008)074006.

The ambiguity in the logarithm of the characteristic function results from the freedom of choosing f:

$$\log\left(\frac{\rho'(\mathbf{K},\mathbf{q})}{\rho'(\mathbf{K},\mathbf{0})}\right) = \log\left(\frac{\rho(\mathbf{K},\mathbf{q})}{\rho(\mathbf{K},\mathbf{0})}\right) + i\left(f(\mathbf{K}+\frac{1}{2}\mathbf{q}) - f(\mathbf{K}-\frac{1}{2}\mathbf{q})\right).$$

Expanding in powers of q_x, q_y, q_z one finds:

$$\mathcal{K}'(r_x, r_y, r_z) = \mathcal{K}'(r_x, r_y, r_z) + (\frac{-i}{2})^{r-1}\frac{\delta^r f(\mathbf{K})}{\delta^{r_x}K_x\delta^{r_y}K_y\delta^{r_z}K_z}\frac{1-(-1)^r}{2}$$

Conclusions I

K. Zalewski, *Phys. Rev.*. **D77**(2008)074006, U.A. Wiedemann and U. Heinz, Phys. Rep. 319(1999)145, P. Danielewicz and S. Pratt, *Phys. Rev. C75*(2007)034907

The even cumulants (r even) can be unambiguously measured, In particular:

- The *HBT* radii, which can be expressed by the $r = 2$ cumulants, are unambiguously measurable.
- The kurtosis matrix which can be expressed by the $r = 4$ cumulants, as in general all the even cumulants, can be unambiguously measured.
- The distribution of $\mathbf{x}_1 - \mathbf{x}_2$ for pairs of points in the interaction region, which is important for the imaging method is unambiguously measurable, because it can be expressed in terms of even cumulants only.

Conclusions II

K. Zalewski, *Phys. Rev.*. **D77**(2008)074006, S. Pratt, Phys. Rev. Lett. 53(1984)1219.

The odd cumulants, which are as important for the determination of the profiles of the homogeneity regions as the even ones, are not measurable. In particular

- The positioning of the centers of the homogeneity regions $\langle \mathbf{x} \rangle$ are unmeasurable.
- The distribution of $\langle \mathbf{x} \rangle(\mathbf{K}) = \nabla f(\mathbf{K})$ is almost unconstrained.
- The (almost arbitrary) shifts of the homogeneity regions are accompanied by deformations
- The uncertainties of the odd cumulants are strongly correlated. E.g.if the $\mathbf{K}$ dependence of $\langle \mathbf{x} \rangle$ is removed by introducing model assumptions, there is no further ambiguity in the determination of the profiles $p(\mathbf{X}|\mathbf{K})$.