Eightfold Way From Dynamics and Confinement in Strongly Coupled Lattice QCD

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OUR LONG STANDING PROGRAM:

GOAL: Try To **FILL In The GAP Between QCD and NUCLEAR PHYSICS**

Starting From FIRST PRINCIPLES (Quarks, Gluons And QCD dynamics): Prove From The Dynamics That

HADRONS And Their BOUND STATES

Are Part Of The ENERGY-MOMENTUM **(E-M)** SPECTRUM

Possibly, UNDERSTAND Better, From Theory, The Nature Of BINDING POTENTIALS

PART OF IT: DONE Already. $2+1$ And $3+1$ Dimensional Models, 2×2, 4×4 Spin Matrices, 1 And 2 Flavors.

FRAMEWORK: IMAGINARY-TIME LATTICE Models Within STRONG COUPLING And FUNCTIONAL INTEGRAL Formulation.

BOUND STATE RESULTS:

BS OBTAINED In A LADDER Approximation, Using Lattice BETHE-SALPETER Equation.

REMARK: The Analysis Here Is Not Trivial Since There Are NO CENTER OF MASS Coordinates On The Lattice.

TREATMENT IS TUNED TO CONTROL CONTRIBUTIONS BEYOND LADDER APPROX.

A LOT STILL TO BE DONE!
Baryon Baryon BS $I = 2, 3$ (3+1), BS With MESONS, $1/r$ CORRECTION To $e^{-m_\pi r}$ IN YUKAWA, EXOTIC STATES, Etc...

TODAY:

Continuing Our Program, We Will Apply Our ANALYTICAL METHODS To ANALYZE The 3+1 Dimensional, 3 Color, 3 Flavor Case, Strongly Coupled Lattice QCD.

OBTAIN The EXACT MESON SPECTRUM

And With A Similar Result For

EXACT BARYON SPECTRUM

OBTAIN THE

GELL'MANN-NE'EMAN EIGHTFOLD WAY

FROM THE DYNAMICS AND SHOW

CONFINEMENT

(I.E., The ONLY SPECTRUM Up To The TWO-MESON THRESHOLD Is Generated By The EIGHTFOLD WAY Particles).

THE MODEL

PARTITION FCT & EXPECTATIONS

$$Z = \int e^{-S(\psi,\bar{\psi},g)}\, d\psi\, d\bar{\psi}\, d\mu(g)\,,$$

$$\langle F\rangle \;=\; \frac{1}{Z}\int F(\bar{\psi},\psi,g)\, e^{-S(\psi,\bar{\psi},g)}\, d\psi\, d\bar{\psi}\, d\mu(g)\,.$$

For ($\hat{}$ = Bar/No-Bar):

- $\widehat{\psi}_{a\alpha f}(u)$ Grassmann Quark Variable At Site u.
- $d\widehat{\psi}$ Associated 'Berezin' Measures.
- $g \in$ SU(3) On Oriented Lattice Bonds.
- $d\mu(g)$ Is Product Measure Of SU(3) Haar Measures.

MODEL ACTION (Wilson's Action)

For $S \equiv S(\psi, \bar{\psi}, g)$

$$S = \frac{\kappa}{2} \sum{}' \bar{\psi}_{a\alpha f}(u)\, \Gamma^{\epsilon e^{\mu}}_{\alpha\beta} (g_{u,u+\epsilon e^{\mu}})_{ab}\, \psi_{b\beta f}(u + \epsilon e^{\mu})$$
$$+ \sum \bar{\psi}_{a\alpha f}(u) \psi_{a\alpha f}(u)$$
$$- \beta \sum_{\mathbf{p}} \chi(g_p)\,.$$

- $\sum$ Is Over $u = (u^0, \vec{u})$, $u^0 \in \mathbb{Z} + \frac{1}{2}$, $\vec{u} \in \mathbb{Z}^3$, $a, b = 1, 2, 3$, $\alpha, \beta = 1, 2, 3, 4$, $f = u, d, s$, And $\sum{}'$, Also Sums Over $\epsilon = \pm 1$ $\mu = 0, 1, 2, 3$.

- $\Gamma^{\pm e^{\mu}} = 1 \pm \gamma^{\mu}$, γ^{μ} = Dirac Spin Matrices.

- $\sum_{\mathbf{p}}$ Is Over **Plaquettes**. χ Is $\Re e$(Character).

- $M \equiv M(m, \kappa) = (m + 2\kappa)\mathbb{I}_{\mathsf{spin}}$ Is Set To $\mathbb{I}_{\mathsf{spin}} \equiv 1$ By Suitably Choosing $m > 0$.

- Hopping Parameter: κ.

- Pure Gauge Coupling: β.

SYMMETRIES Of The Action

- **GAUGE INVARIANCE:**

  For $x \in \mathbb{Z}_o^4 = (\mathbb{Z} + \frac{1}{2}) \times \mathbb{Z}^3$ and $h(x) \in$ SU$_c(3)$,

  $$\psi(x) \mapsto h(x)\psi(x), \quad \bar{\psi}(x) \mapsto \bar{\psi}(x)[h(x)]^{-1},$$
  $$g_{x+e^\mu,x} \mapsto h(x+e^\mu) \; g_{x+e^\mu,x} \; [h(x)]^{-1}.$$

- **FLAVOR** Or **ISOSPIN SYMMETRY:** GLOBAL SU$(3)_f$.

- **FOR SPIN:** NO CONTINUOUS Rotation Symmetry. ONLY Discrete $\pi/2$ Spatial Rotations.

  At $\kappa = 0$, RECOVER SU(2) $\oplus$ SU(2) SPIN STRUCTURE.

  WHEN $\kappa \neq 0$, We Have A PARTIAL RESTORATION Of Continuous Symmetry FOR IMPROPER ZERO SPATIAL MOMENTUM STATES.

  THIS IS WHAT WE USE TO TALK ABOUT SPIN!

This Is NOT The FIRST TIME This Problem Is Treated In The Literature!

In The 80's Some Papers (Smit, Hoeck, ...) Were Devoted To Analyze This Problem.

Many Results Emerge In Our Treatment That Were Not Obtained Previously:

- Connection of SINGULARITIES OF CORRELATIONS And E-M SPECTRUM.
  - MAIN TOOL: Spectral Representation For Associated Correlation Functions.

- EXACT MASSES as CONVERGENT EXPANSIONS In The Hopping Parameter κ And Pure Gauge Coupling β.

- GOOD CONTROL Of The DECAY PROPERTIES Of Correlations.

– Composite GAUGE INVARIANT FIELDS For Free.

– UPPER GAP PROPERTY (Isolated Dispersion Curves In The E-M Spectrum).

  * MAIN TOOL: The Decoupling Of Hyperplane Method.

– CONFINEMENT In The Hilbert Space Up The Two-Meson Energy Threshold.

  * MAIN TOOL: Euclidean Subtraction Method For Correlations.

AND The One-Particle Spectrum, As It Is Done HERE, IS A NECESSARY STEP To Go Up In Spectrum and Obtain Two-Particle Spectrum.

DOMAIN: STRONG COUPLING REGIME

Small HOPPING PARAMETER κ,

$0 < \kappa \ll 1$.

Much Smaller PURE GAUGE COUPLING β,

$0 < \beta \ll \kappa$.

Such That:

- FAR From SCALING LIMIT But Can Manage It.

- For $\beta << \kappa$ The Low-Lying Spectrum Consists Solely Of The Eightfold Way Particles (No Glueballs).

- Also, CONFINEMENT Shows Up In This Way.

MESONS AFN, MO'C & PAFdV - PRD and JMP 2008

1. EXISTENCE OF 36 MESONS: (Masses $\approx -2 \ln \kappa$). Manifested by Isolated Dispersion Curves in the EM Spectrum $w(\vec{p}) \equiv w(p^1, p^2, p^3)$, $p^{i=1,2,3} \in (-\pi, \pi]$. (Isolated Up To Near The Meson-Meson Threshold $\approx -4 \ln \kappa$).

2. MESON EIGHTFOLD WAY: Consider QUANTUM NUMBERS ISOSPIN, HYPERCHARGE and C_2 (QUADRATIC CASIMIR FOR $\mathrm{SU}_f(3)$)

The 36 Mesons Can Be Grouped Into FOUR ISOSPIN NONETS: The Pseudo-Scalar Mesons and The Vector Mesons.

There are: 9 Pseudo-Scalar Mesons With Total Spin $J = 0$ And 27 Vector Mesons With $J = 1$.

Each NONET: Decomposed Into A Singlet ($C_2 = 0$) And Octet ($C_2 = 3$) Isospin.

They Have The MASS

$$M(\kappa, \beta = 0) = -2 \ln \kappa - 3\kappa^2/2 + \kappa^4 r(\kappa),$$

With $r(\kappa)$ Analytic And $r(0) \neq 0$ With $M(\kappa, \beta) + 2 \ln \kappa$ jointly analytic in κ and β.

For Zero Momentum States, The Masses Are Independent Of J_z. Therefore, As For The PSEUDO-SCALAR MESONS, **ALL** Vector Mesons Have The SAME MASS.

3. MESON MASS SPLITTING: Between Vector And Pseudo-Scalar Mesons

$$[r_p(\kappa) - r_v(\kappa)]\kappa^4 = 2\kappa^4 + \mathcal{O}(\kappa^6).$$

And The Splitting Persists For $\beta \neq 0$.

4. MESON DISPERSION RELATIONS:

For $\vec{p}_\ell^{\,2} = \sum_{j=1}^{3} 2(1 - \cos p^j)$:

$$\begin{aligned} w_c(\kappa, \vec{p}) = & -2\ln\kappa - 3\kappa^2/2 + (1/4)\kappa^2\vec{p}_\ell^{\,2} \\ & +\kappa^4 r_c(\kappa, \vec{p}), \quad c = p, v \end{aligned}$$

$|r_c(\kappa, \vec{p})| \leq$ const And $r_p(\kappa, \vec{p})$ Is Jointly Analytic In κ And p^j For $|\kappa|$, $|\Im m\, p^j|$ Small.

The various $w_v(\kappa, \vec{p})$ May Depend On $|J_z|$.

5. MESON SPECTRAL RESULTS: In $\mathcal{H}_e$ (EVEN SUBSPACE Of Quantum Mechanical Hilbert Space $\mathcal{H}$) A SUBTRACTION METHOD PROVES CONFINEMENT UP To NEAR The TWO-MESON THRESHOLD.

FEYNMAN-KAC (F-K) FORMULA

Consider $G(\psi, \bar{\psi}, g)$, $F(\psi, \bar{\psi}, g)$, Supported On $u^0 = 1/2$. For $x^0 > 0$,

$$(G, \check{T}_0^{x^0} \check{T}_1^{x^1} \check{T}_2^{x^2} \check{T}_3^{x^3} F)_{\mathcal{H}}$$
$$= \langle [T_0^{x^0} T_1^{x^1} T_2^{x^2} T_3^{x^3} F] \Theta G \rangle \, .$$

- Θ Is An Anti-Linear Operator Which Involves Time-Reflection.

- $\check{T}_\mu^{x^\mu}$ Are Operators In $\mathcal{H}$. $\check{T}_0^2 = e^{-2H}$ (H Is The Self-Adjoint Energy Operator) And $\check{T}_{j=1,2,3} = e^{iP_j}$ (P_j Is The Self-Adjoint Momentum Operator).

- $T_\mu^{x^\mu}$ Translations Of Grassmann Fcts In Time $\mu = 0$ And Space $\mu = 1, 2, 3$.

- LHS: INNER PRODUCTS In $\mathcal{H}$.

- RHS: STATISTICAL MECHANICAL AVERAGES (CORRELATIONS).

  Taking Fourier Transform of The F-K Formula Implies Relations Between COMPLEX MOMENTUM SINGULARITIES And The E-M SPECTRUM.

MESON STATES

HYPERPLANE EXPANSION: $\kappa \to \kappa_p$

In Hopping Term Between $x^0 = p$ And $y^0 = p + 1$.

$$\mathcal{G}_{ML}(x,y) \equiv \begin{cases} \langle \Theta M(x) L(y) \rangle \,, & u^0 \leq v^0 \\ \langle M(x) \Theta L(y) \rangle^* \,, & u^0 > v^0 \end{cases} .$$

Intuitively, For Each Vanishing κ_p Derivative And For Each Intermediate Hyperplane We Pick Up A Factor Of κ For The Exponential Decay Of The Truncated Correlation.

For L And $M \in \mathcal{H}_e$ We Get $\mathcal{G}^{(r)}_{LM}(x,y) = 0$ $(r = 0, 1)$, For The 0th And 1st κ_p Derivative.

For The SECOND κ_p Derivative

$$\mathcal{G}^{(2)}_{LM}(x,y) = \sum_{\vec{\gamma},\vec{g}} \sum_{\vec{w}} \mathcal{G}^{(0)}_{L\bar{\mathcal{M}}_{\vec{\gamma}\vec{g}}}(x,(p,\vec{w})) \times \mathcal{G}^{(0)}_{\bar{\mathcal{M}}_{\vec{\gamma}\vec{g}}M}((p+1,\vec{w}),y)\,,$$

Notation $\vec{\gamma} \equiv (\gamma^{\ell}, \gamma^{u})$, $g \equiv (g_1, g_2)$:

$$\bar{\mathcal{M}}_{\vec{\gamma}\vec{g}}(x) = \frac{1}{\sqrt{3}} \bar{\psi}_{a\gamma^{\ell}g_1}(x)\psi_{a\gamma^{u}g_2}(x).$$

Where For SPIN Indices: $\gamma^{\ell} =$ LOWER $= 3,4$, $\gamma^{u} =$ UPPER $= 1,2$.

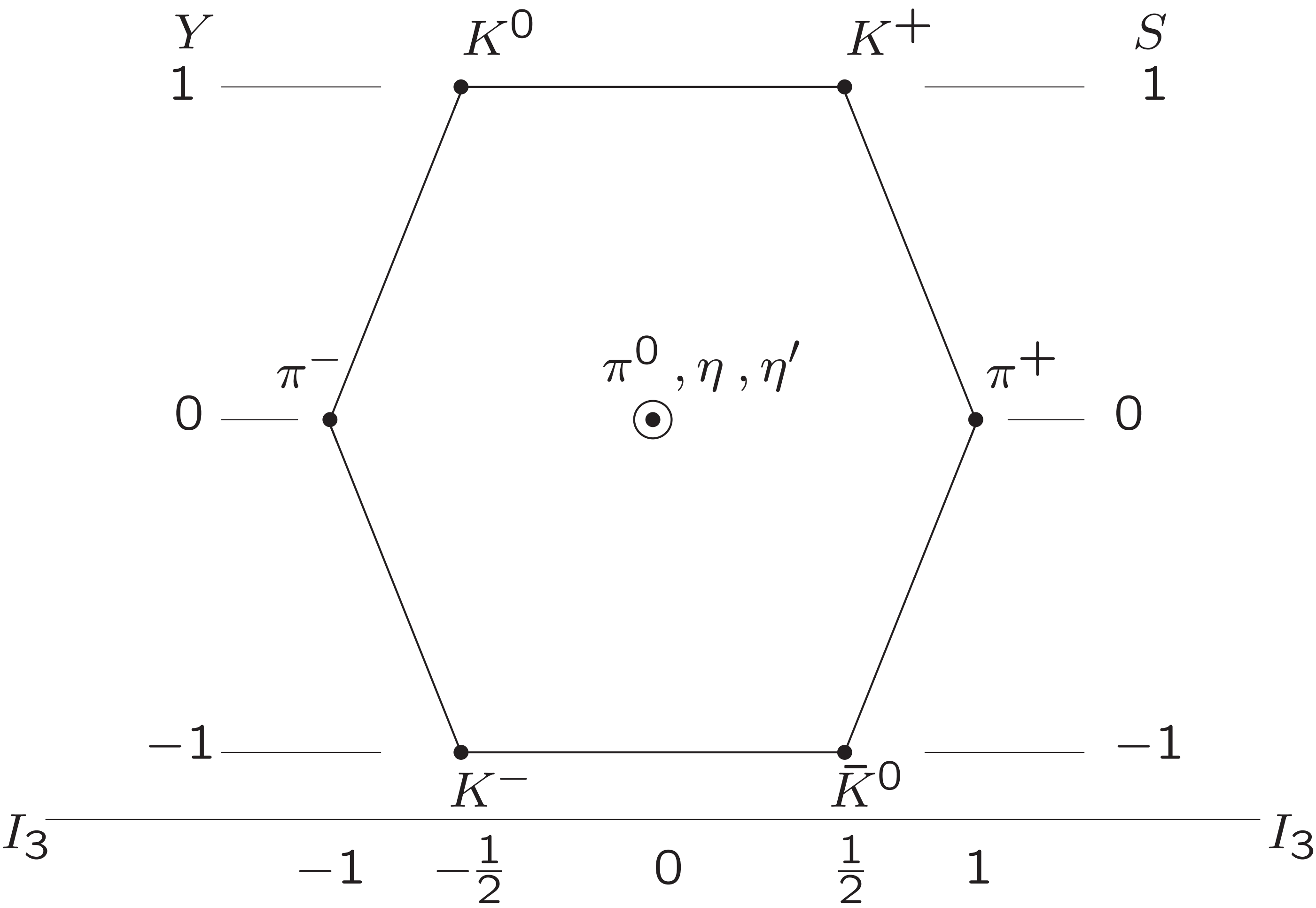
For CLOSURE (Same Correlations On The RHS/LHS):

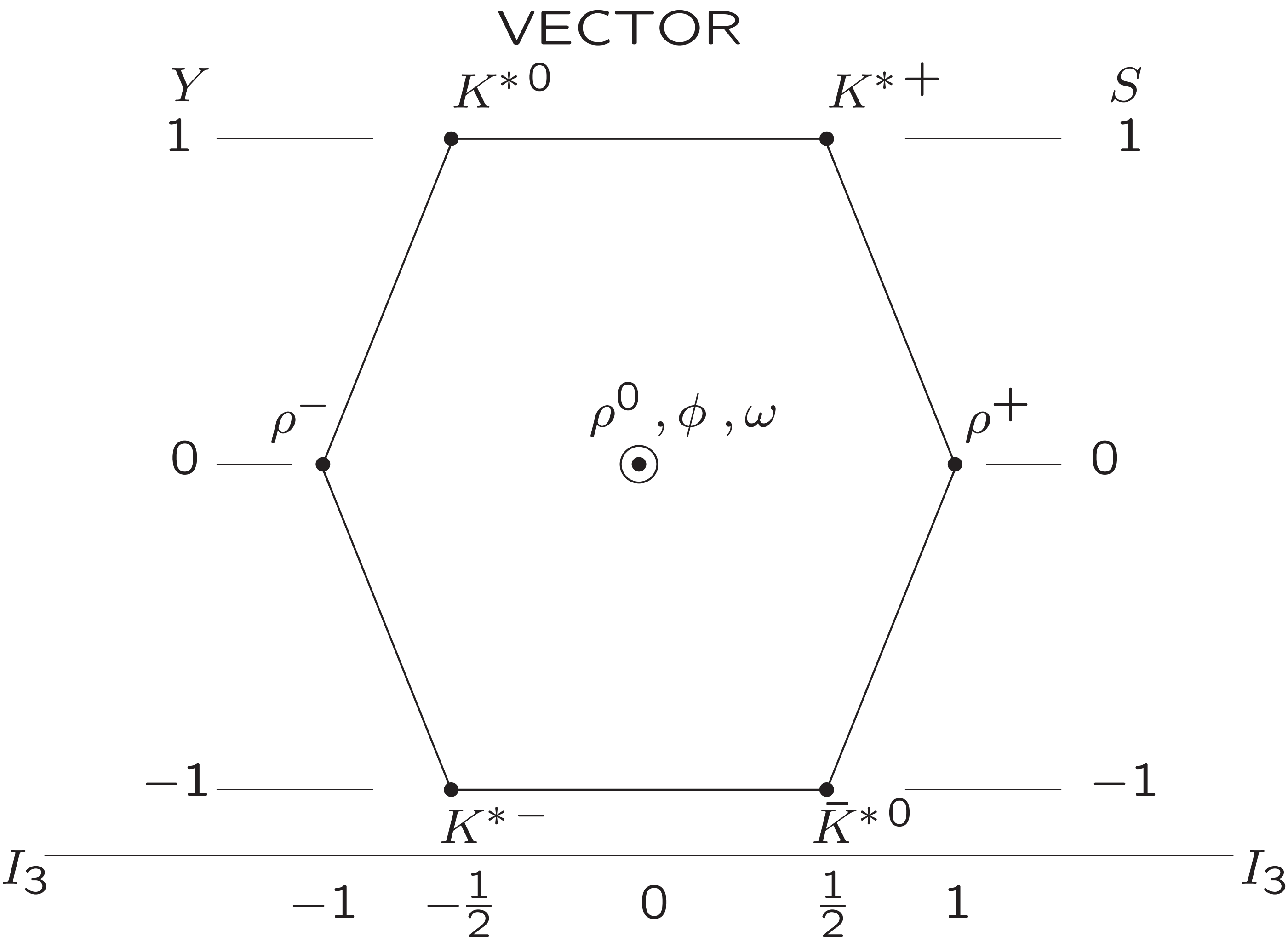
$L = \bar{\mathcal{M}}_{\vec{\alpha}\vec{f}}(x)$ and $M = \bar{\mathcal{M}}_{\vec{\beta}\vec{h}}(x)$.

This Is How The MESON Fields Emerge NATURALLY As Gauge Invariant Fields!

SPECTRAL REPRESENTATION +
DECOUPLING OF HYPERPLANE
GIVE:

PSEUDO-SCALAR





VECTOR
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K*0
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1
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ρ0 , φ , ω
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K*−
K̄*0
I3
I3
−1
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0
1/2
1

Subtraction Method For Correlations

Define The Subtracted Correlation $(L \in \mathcal{H}_e)$

$$\mathcal{F} \equiv \mathcal{G}_{LL} - \mathcal{G}_{L\bar{\mathcal{M}}} \mathcal{G}^{-1}_{\bar{\mathcal{M}}\bar{\mathcal{M}}} \mathcal{G}_{\bar{\mathcal{M}}L}$$

Such That, By The DECOUPLING OF HYPERPLANE METHOD :

- $\mathcal{F}$ Has A FASTER DECAY Than $\mathcal{G}_{LL}$, I.E., The Singularities Of The FIRST And SECOND Term On The RHS Are Canceled.

- $\mathcal{G}_{L\bar{\mathcal{M}}}$ And $\mathcal{G}_{\bar{\mathcal{M}}L}$ Have The Same Spectral Representation As The Two-Point Meson Correlation.

- Hence, The Only Singularities Of The Fourier Transform Of $\mathcal{G}_{LL}$ (Up To $\approx -4 \ln \kappa$) Are Given By The Eightfold Way Particles. And, CONFINEMENT IS THUS PROVEN.

FINAL COMMENT: As We Know How To Obtain BOUND STATE SPECTRUM, We Are Currently Investigating

EXOTIC STATES:

– TETRAQUARKS.

– PENTAQUARKS.

– HYBRID MESONS And BARYONS.

EIGHTFOLD WAY PUBLICATIONS:

(1)A. Francisco Neto, Michael O'Carroll And P.A. Faria da Veiga, *Dynamical Eightfold Way Mesons in Strongly Coupled Lattice QCD*, Physical Review D77, 054503 (2008).

(2)A. Francisco Neto, Michael O'Carroll And P.A. Faria da Veiga, *Mesonic eightfold way from dynamics and confinement in strongly coupled lattice quantum chromodynamics*, J. Math. Phys. 49, 072301 (2008).

(3)P.A. Faria da Veiga And Michael O'Carroll, *Eightfold Way From Dynamical First Principles in Strongly Coupled Lattice QCD*, J. Math. Phys. 49, 042303 (2008).

RELATED PUBLICATIONS:

(1) *Understanding Baryons From First Principles*, **Phys. Rev. D67, 017501 (2003);**

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(11) *Hadron-Hadron Bound States*, AIP Conference Proceedings, J.E.Ribeiro ed., QCHS-7, Açores, Portugal, (2006);

ALL ABOVE PAPERS ARE AVAILABLE AT THE SITE:

www.icmc.usp.br/∼veiga

(12) *Meson-Baryon Bound States in a 2 + 1-dimensional Strongly Coupled Lattice QCD Model, Phys. Rev. D70, 037502 (2004);*

(13) *A Meson-Baryon Bound State in a 2 + 1 Lattice QCD Model With Two Flavors and Strong Coupling*, Phys. Scr. 75, 484 (2007).