

Finite-top-mass effects in NNLO Higgs production

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Outline

The inclusive gluon-gluon fusion cross section

Computation of the high energy limit

Improvement of the fixed-order results

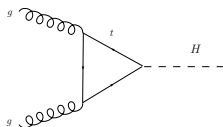
Conclusions and Outlook

Searching the Higgs boson at the LHC

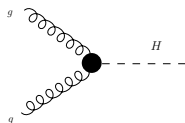
- ▶ The Higgs boson is the missing particle of the SM
- ▶ Its discovery is the main reason the LHC has been built for
- ▶ The main production channel is gluon-gluon fusion via a top loop
- ▶ We study finite-top-mass effects on the inclusive cross section

The state of the art (QCD)

- ▶ **NLO** (large contribution) [Spira et al. 1995]



- ▶ The dominant contribution does not resolve the effective ggH vertex:
 $m_t \rightarrow \infty$ limit (one less loop)
- ▶ **NLO** [Spira et al. 1991; Dawson, 1991]
- ▶ **NNLO** [Anastasiou and Melnikov, 2002; Harlander and Kilgore, 2002; Ravindran, Smith and van Neerven, 2003]



The high energy behaviour

- ▶ The heavy top approximation fails in the hard limit $\hat{s} \rightarrow \infty$:

$$\hat{\sigma} \underset{\hat{s} \rightarrow \infty}{\sim} \begin{cases} \alpha_s^2 \sum_{k=1}^{\infty} \alpha_s^k \ln^{2k-1} \left(\frac{\hat{s}}{m_t^2} \right) & \text{pointlike: } m_t \rightarrow \infty \\ \alpha_s^2 \sum_{k=1}^{\infty} \alpha_s^k \ln^{k-1} \left(\frac{\hat{s}}{m_t^2} \right) & \text{resolved: finite } m_t \end{cases}$$

- ▶ In the pointlike approximation there are **double logs** [Hautmann, 2002]
- ▶ In the finite m_t case the high energy behaviour is softened by a form factor
- ▶ Using high energy resummation techniques we compute the coefficients of the **single logs**
- ▶ We use the high energy behaviour of the exact cross section to improve the infinite m_t results

Some definitions

- ▶ We write the partonic cross section as:

$$\begin{aligned}\hat{\sigma}_{gg}(\alpha_s; \tau; y_t, m_H^2) &= \sigma_0(y_t) C(\alpha_s(m_H^2), \tau, y_t) \\ C(\alpha_s(m_H^2), \tau, y_t) &= \delta(1 - \tau) + \frac{\alpha_s(m_H^2)}{\pi} C^{(1)}(\tau, y_t) \\ &\quad + \left(\frac{\alpha_s(m_H^2)}{\pi}\right)^2 C^{(2)}(\tau, y_t) + \dots\end{aligned}$$

- ▶ $\sigma_0\delta(1 - \tau)$ is the LO cross section
- ▶ $\tau = \frac{m_H^2}{\hat{s}}$ and $y_t = \frac{m_t^2}{m_H^2}$
- ▶ We consider Mellin moments of the coefficient function:

$$C(\alpha_s(m_H^2), N, y_t) = \int_0^1 d\tau \tau^{N-1} C(\alpha_s(m_H^2), \tau, y_t)$$

- ▶ The leading contribution at **small** τ is given order by order in α_s by the highest **pole** in N

High energy factorisation

Leading small N singularities: *high energy factorisation theorem*:

- ▶ LO $gg \rightarrow H$ cross section σ_{off} with off-shell gluons with virtualities ξ_1, ξ_2
- ▶ Mellin moments:

$$h(N, M_1, M_2) = M_1 M_2 \int_0^\infty d\xi_1 \int_0^\infty d\xi_2 \xi_1^{M_1-1} \xi_2^{M_2-1} \int_0^1 d\tau \tau^{N-1} \sigma_{off}$$

- ▶ Leading small N singularities of the coefficient function:

$$h(0, \gamma^+(N), \gamma^+(N))$$

- ▶ $h(0, M_1, M_2)$ cannot be computed analytically because of the complexity of σ_{off}
- ▶ Expansion in M_1, M_2 and numerical evaluation of the integrals
- ▶ Coefficients computed up to the 4th order in M_i (N^4 LO) in view of resummation

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Results

- ▶ Remember that:

$$C(\alpha_s(m_H^2), N, y_t) = 1 + \frac{\alpha_s(m_H^2)}{\pi} C^{(1)}(N, y_t) + \left(\frac{\alpha_s(m_H^2)}{\pi}\right)^2 C^{(2)}(N, y_t) + \dots$$

- ▶ We compute:

$$C^{(1)}(N, y_t) = C^{(1)}(y_t) \frac{C_A}{N} [1 + O(N)]$$

$$C^{(2)}(N, y_t) = C^{(2)}(y_t) \frac{C_A^2}{N^2} [1 + O(N)]$$

- ▶ Upon Mellin inversion:

$$C^{(1)}(\tau, y_t) = C^{(1)}(y_t) C_A + O(\tau)$$

$$C^{(2)}(\tau, y_t) = -C^{(2)}(y_t) C_A^2 \ln \tau + O(\tau^0)$$

- ▶ $C^{(1)}(y_t)$ agrees with the small τ limit of the full NLO computation
- ▶ The NNLO coefficient $C^{(2)}(y_t)$ is our main result.

Matching

- ▶ Our computation only determines the leading small τ behaviour of the coefficient function $C(\tau, y_t)$:

$$C(\tau \rightarrow 0, y_t)$$

- ▶ It has to be matched with the $m_t \rightarrow \infty$ result at large τ

$$C^{\text{app.}}(\tau, y_t) \approx C(\tau, \infty) + \left[C(\tau \rightarrow 0, y_t) - \lim_{\tau \rightarrow 0} C(\tau, \infty) \right] T(\tau, \tau_0)$$

- ▶ Step function $T(\tau, \tau_0) = \theta(\tau_0 - \tau)$
- ▶ Smooth function $T(\tau, \tau_0) = \frac{1}{2} \left[1 + \tanh\left(\frac{\tau_0 - \tau}{\omega}\right) \right]$
- ▶ How do we choose τ_0 ?

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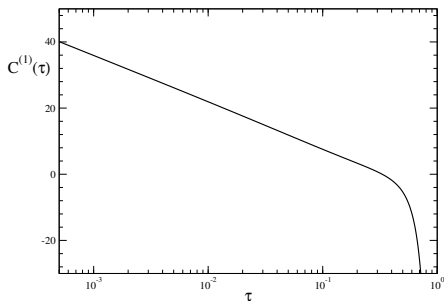
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NLO with $m_H = 130\text{GeV}$

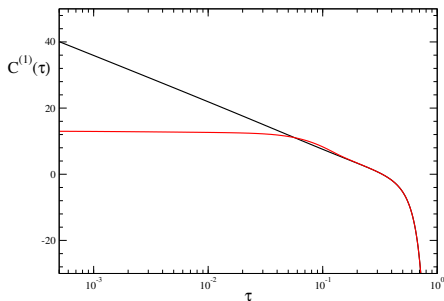
- ▶ $m_t \rightarrow \infty$ case $C^{(1)}(\tau, \infty)$
- ▶ Spurious logarithmic growth at small τ



The full NLO computation is known, so we can use it as a testing ground

NLO with $m_H = 130\text{GeV}$

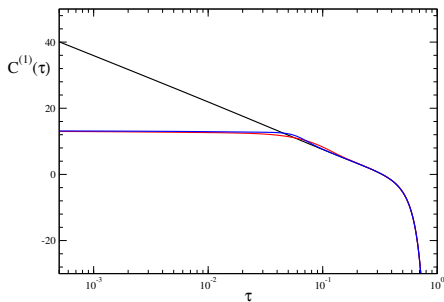
- ▶ Exact case $C^{(1)}(\tau, y_t)$, with $m_t = 170.9\text{GeV}$
- ▶ Correct small τ behaviour



Matching point at the intersection of $m_t \rightarrow \infty$ with the asymptotic curve

NLO with $m_H = 130 \text{ GeV}$

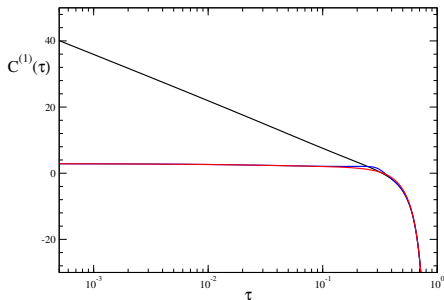
$$C^{(1),\text{app.}}(\tau, y_t) \approx C^{(1)}(\tau, \infty) + \left[C^{(1)}(y_t) C_A - \lim_{\tau \rightarrow 0} C^{(1)}(\tau, \infty) \right] T(\tau, \tau_0)$$



Very small discrepancy between approximate and exact results

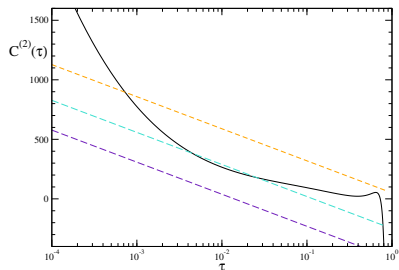
NLO with $m_H = 280\text{GeV}$

For a heavier Higgs boson the spurious growth sets in at larger values of τ



- ▶ $m_t \rightarrow \infty$
- ▶ exact
- ▶ approximate

NNLO with $m_H = 130\text{GeV}$ (I)

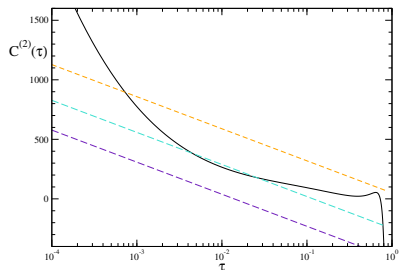


$$C^{(2),\text{app.}}(\tau, y_t) \approx C^{(2)}(\tau, \infty) + \left[-C^{(2)}(y_t) C_A^2 \ln \tau + C_0^{(2)}(y_t) - \lim_{\tau \rightarrow 0} C^{(2)}(\tau, \infty) \right] T(\tau, \tau_0)$$

- ▶ Spurious $\ln^3 \tau$ growth to be replaced with $\ln \tau$
- ▶ The calculation fixes the slope but not the constant
- ▶ How do we choose τ_0 ?

How do we deal with the constant ?

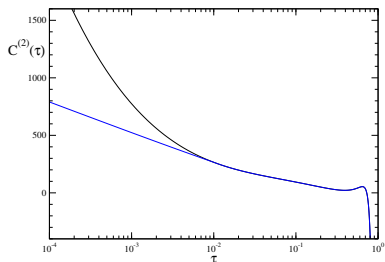
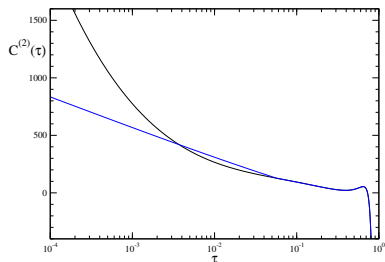
NNLO with $m_H = 130\text{GeV}$ (I)



$$C^{(2),\text{app.}}(\tau, y_t) \approx C^{(2)}(\tau, \infty) + \left[-C^{(2)}(y_t) C_A^2 \ln \tau + C_0^{(2)}(y_t) - \lim_{\tau \rightarrow 0} C^{(2)}(\tau, \infty) \right] T(\tau, \tau_0)$$

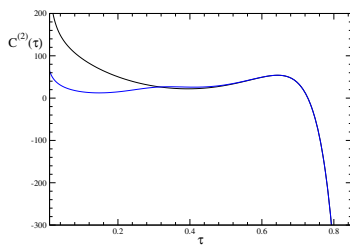
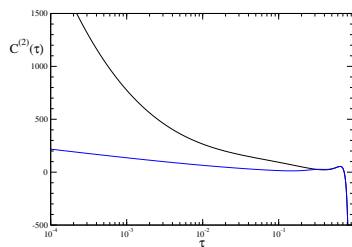
- ▶ Spurious $\ln^3 \tau$ growth to be replaced with $\ln \tau$
- ▶ The calculation fixes the slope but not the constant
- ▶ **How do we choose τ_0 ?** **How do we deal with the constant ?**

NNLO with $m_H = 130\text{GeV}$ (II)



- ▶ Two procedures to determine τ_0 :
same as NLO (left) or matching the slopes (right)
- ▶ The two methods give similar curves
- ▶ In both cases the constant is adjusted requiring continuity

NNLO with $m_H = 280\text{GeV}$



- $m_t \rightarrow \infty$ and improved partonic cross section for $m_H = 280\text{GeV}$

NNLO hadronic cross section

- ▶ Hadronic cross section $\sigma_{gg}(\tau_h; y_t, m_H^2)$ computed with MRST2002 gluon

$$K_{gg}(\tau_h; y_t, m_H^2) = \sigma_{gg}(\tau_h; y_t, m_H^2) / \sigma_{gg}^0(\tau_h; y_t, m_H^2)$$

$m_H = 130$ GeV	K_{gg}	$m_H = 280$ GeV	K_{gg}
pointlike	2.140	pointlike	2.420
approximate	2.136	approximate	2.395

- ▶ Small- τ finite- m_t effects on K are $\sim 1\%$ for $m_H \leq 280$ GeV

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approximate	2.136	approximate	2.395

- ▶ Small- τ finite- m_t effects on K are $\sim 1\%$ for $m_H \leq 280$ GeV

Conclusions

- ▶ We have discussed the inclusive cross section for Higgs boson production via gluon-gluon fusion
- ▶ Thanks to high energy factorisation we have computed the leading high energy (small τ) singularities of the partonic cross section
- ▶ We have constructed an approximation to the cross section matching the exact small τ behaviour to the infinite m_t calculation
- ▶ Hard-effects in the partonic coefficient function are under control at NNLO (and beyond)
- ▶ The impact on the inclusive cross section is not large $\sim 1\%$

Outlook

- ▶ The rapidity distribution may have more pronounced finite- m_t effects
- ▶ The resummation of the small τ logarithms is work in progress (the first four orders have been computed)
- ▶ The combination of high energy ($\tau \rightarrow 0$) and soft contributions ($\tau \rightarrow 1$) will provide information about unknown higher order corrections