

Precise α_s from τ Decays(*)

M. Davier, S. Descotes-Genon, A. Hoecker, B. Malaescu, and Z. Zhang

QCD Montpellier, July 2008

(*) arxiv:0803.0979;
To be published in EPJ C

Outline

Motivation:

- ALEPH τ spectral functions with improvements of branching ratios (BABAR and Belle)
- Better knowledge of perturbative series (P. Baikov, et al.)

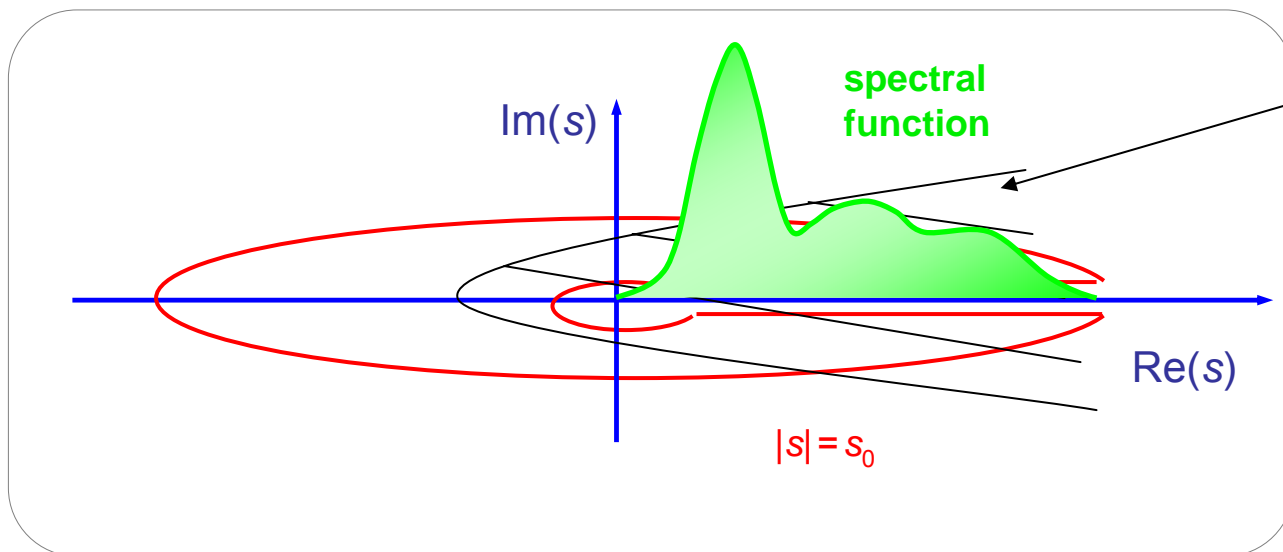
- Theoretical Framework
- Tests of Integration Methods
- Impact of Quark-Hadron Duality Violation
- Spectral Moments and Fit Results
- Test of Asymptotic Freedom
- Conclusions

Theoretical Prediction of R_{τ}

$$R_{\tau,U}(s_0) = 12\pi S_{EW} \int_0^{s_0} \frac{ds}{s_0} \left(1 - \frac{s}{s_0}\right)^2 \left[\left(1 + 2\frac{s}{s_0}\right) \text{Im}\Pi_U^{(1)}(s + i\varepsilon) + \text{Im}\Pi_U^{(0)}(s + i\varepsilon) \right]$$

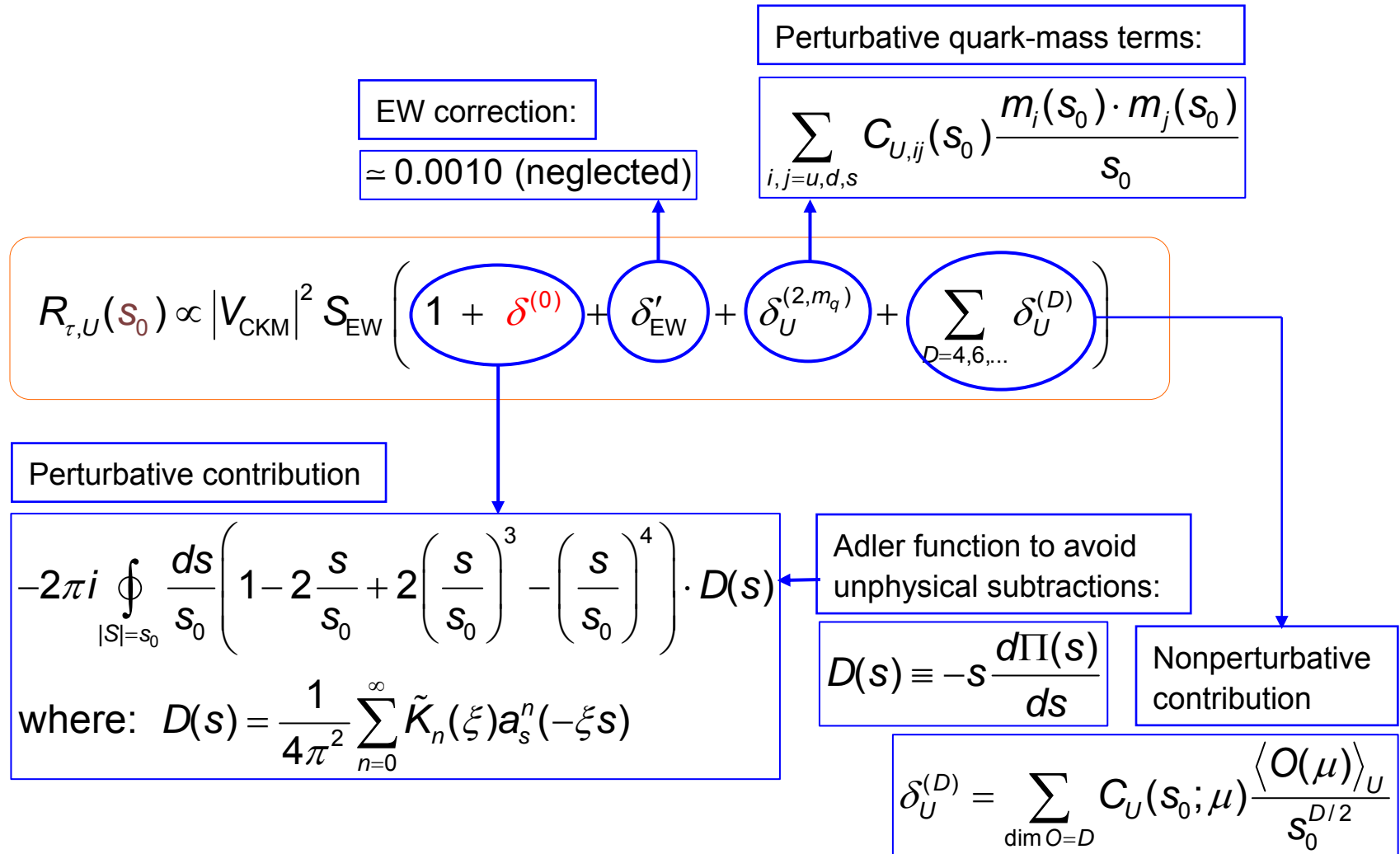
- Optical Theorem: $\text{Im}\Pi_{\bar{u}d(s),V/A}^{(1,0)} = \frac{1}{2\pi} v_1/a_{1,0}(s)$
- Problem: $\text{Im}\Pi_{V/A}^{(j)}(s)$ contains hadronic physics that cannot be predicted by QCD in this region of the real axis
- However, owing to the analyticity of $\Pi(s)$, one can use Cauchy's theorem:

$$R(s_0) \propto \int_0^{s_0} ds w(s) \text{Im}\Pi(s + i\varepsilon) \Leftrightarrow -\frac{1}{2i} \oint_{|s|=s_0} ds w(s) \Pi(s)$$



τ and QCD: The Operator Product Expansion

- Full theoretical ansatz, including nonperturbative operators via the OPE:
(in the following: $a_s = \alpha_s/\pi$)



The Perturbative Prediction

- Perturbative prediction of Adler function given to N³LO

$$\delta^{(0)} = \sum_n \tilde{K}_n(\xi) A^{(n)}(a_s)$$

Perturbative coefficients of Adler function series, known to n=4 ($K_4 \approx 49$)

P. Baikov, et al.,
arxiv:0801.1821[hep-ph]

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} d\varphi (1 + 2e^{i\varphi} - 2e^{i3\varphi} - e^{i4\varphi}) \cdot a_s^n (\xi s_0 e^{i\varphi})$$

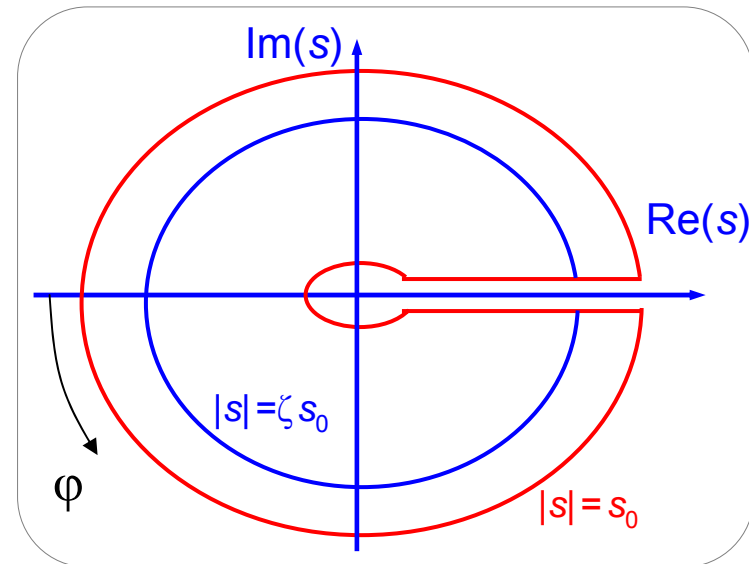
- s dependence of a_s driven by running:

$$\frac{da_s}{d \ln s} = \beta(a_s) = -a_s^2 \sum_{n=0}^{\infty} \beta_n a_s^n$$

RGE β -function,
known to $n=3$

In practice, use Taylor development in $\eta = \ln(s/s_0)$

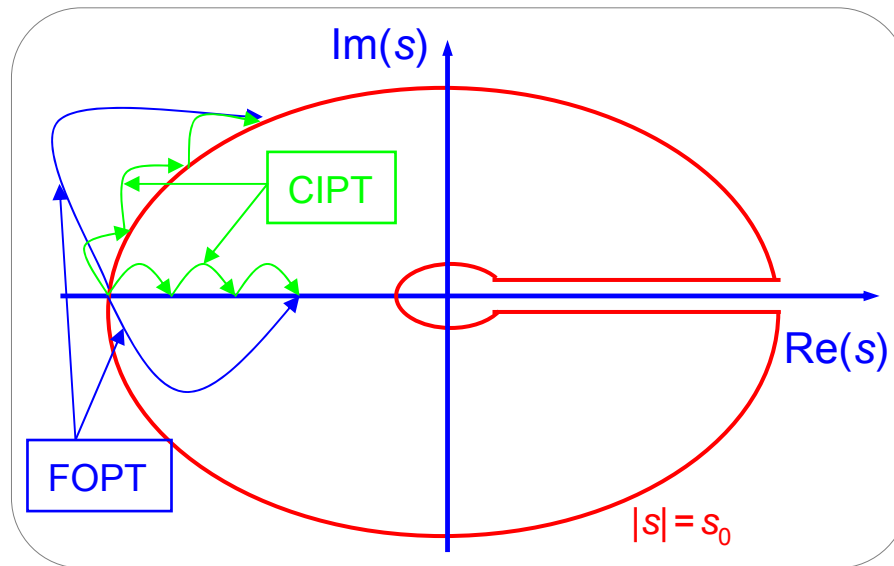
$$a_s(s) = a_s(s_0) - \beta_0 \eta a_s^2(s_0) + \dots$$



- How to compute the integral in the complex plane?
- How to perform a scale transformation?

Numerical Methods

- CIPT: at each step use Taylor series to compute $a_s(s)$ from the value found at the previous step
- FOPT: fixed order Taylor expansion around the physical value $a_s(s_0)$ and the integration result is cut at the same order in $a_s(s_0)$



Remarks:

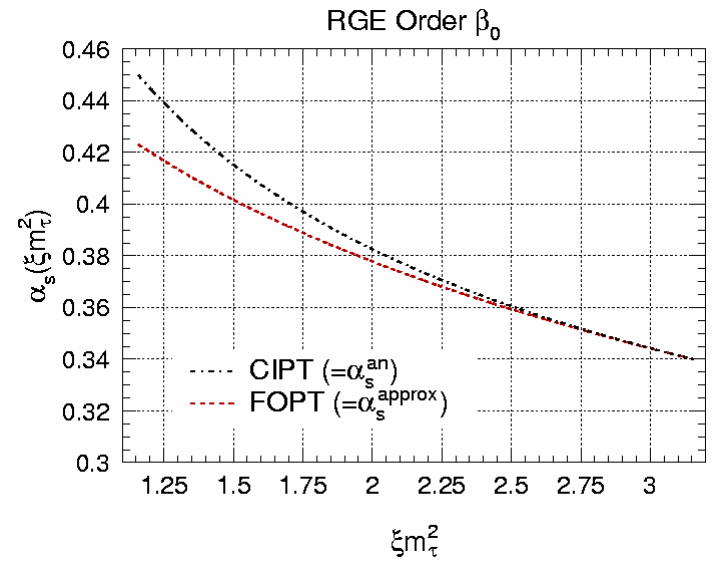
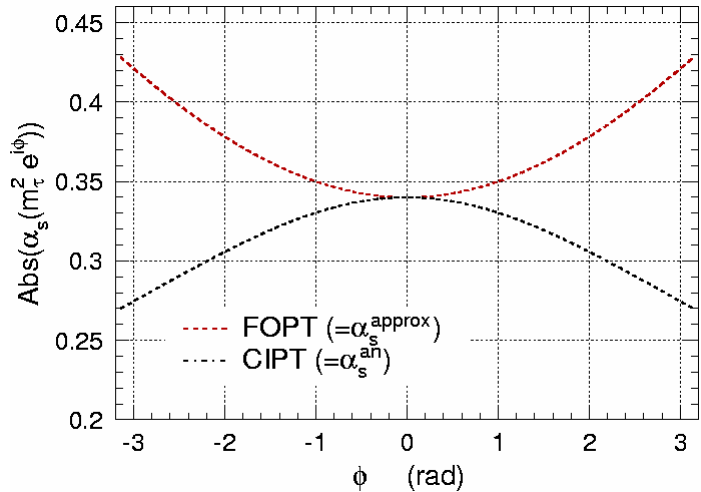
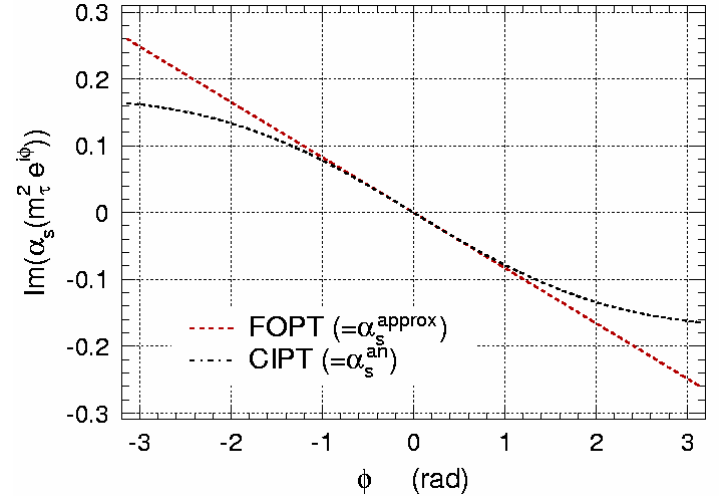
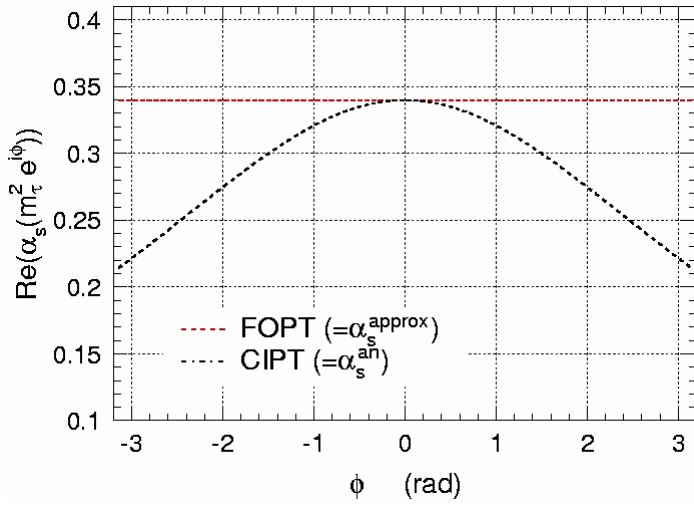
- Potential problem for FOPT due to the use of Taylor series for large $|\eta|$
- Avoided by CIPT (use small steps)
- Analytically, the FOPT result can also be obtained by making small steps, with a fixed order cut of the result at each step, BUT the RGE is modified at each step! ₆

RGE Order β_0

CIPT

FOPT

$$a_s^{an}(s) = \frac{a_s(s_0)}{1 + \beta_0 \cdot a_s(s_0) \cdot \ln\left(\frac{s}{s_0}\right)} \approx a_s(s_0) - \beta_0 \cdot (a_s(s_0))^2 \cdot \ln\left(\frac{s}{s_0}\right)$$

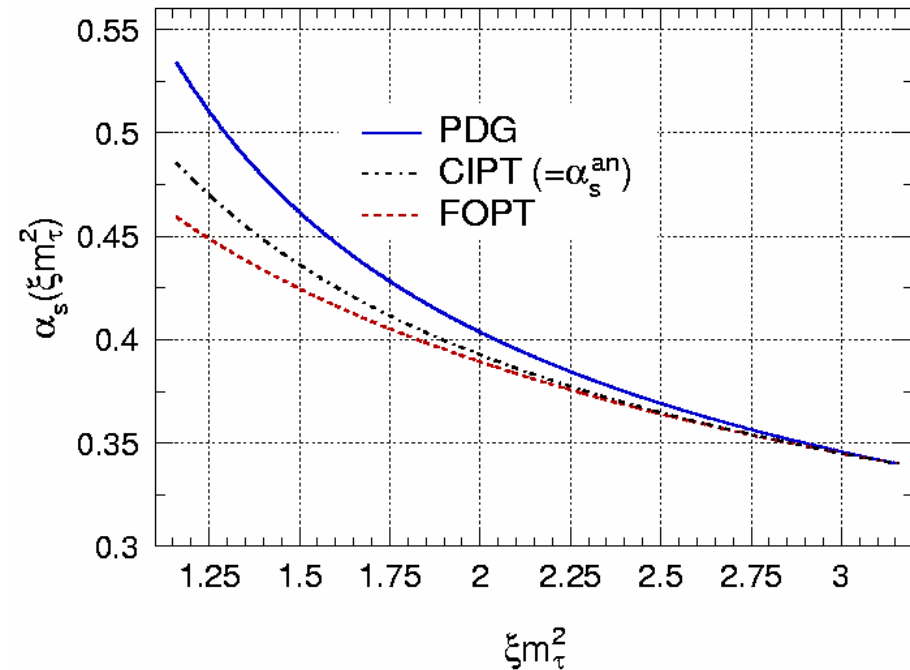


RGE Order $\beta_{1,2}$

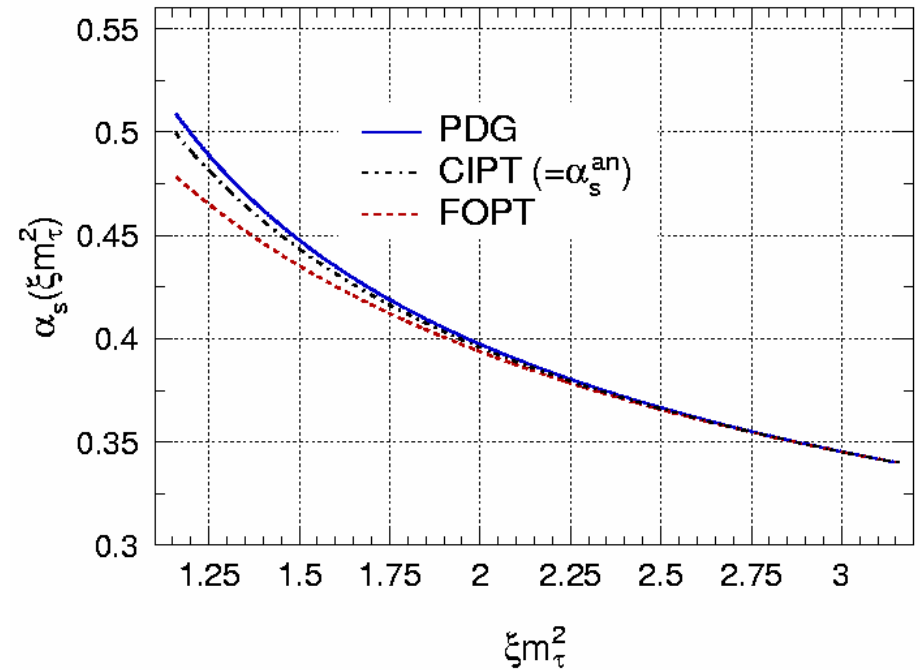
$$\int_{a_s(s_0)}^{a_s(s)} \frac{da}{\beta(a)} = \frac{1}{a \cdot \beta_0} \Big|_{a_s(s_0)}^{a_s(s)} + \frac{\beta_1}{\beta_0^2} \ln \left[\frac{a_s(s) \cdot (\beta_0 + a_s(s_0) \cdot \beta_1)}{a_s(s_0) \cdot (\beta_0 + a_s(s) \cdot \beta_1)} \right] = \ln \frac{s}{s_0} \rightarrow \alpha_s^{an}(s)$$

$$\alpha_s^{PDG}(s) = \frac{1}{\beta_0 \cdot \ln\left(\frac{s}{\Lambda^2}\right)} \left(1 - \frac{\beta_1}{\beta_0^2} \cdot \frac{\ln\left(\ln\left(\frac{s}{\Lambda^2}\right)\right)}{\ln\left(\frac{s}{\Lambda^2}\right)} + O\left(\frac{1}{\ln^2\left(\frac{s}{\Lambda^2}\right)}\right) \right)$$

RGE Order β_1



RGE Order β_2



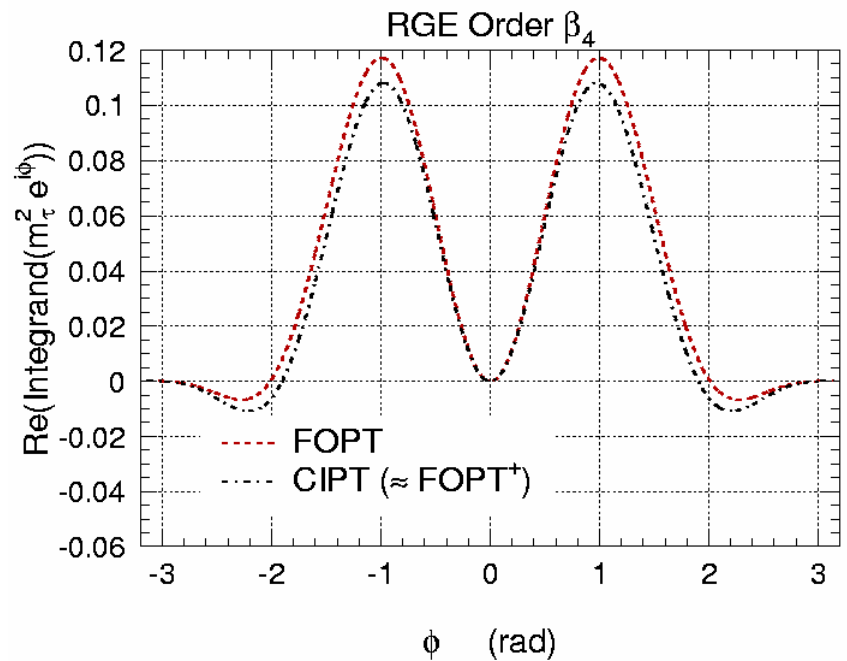
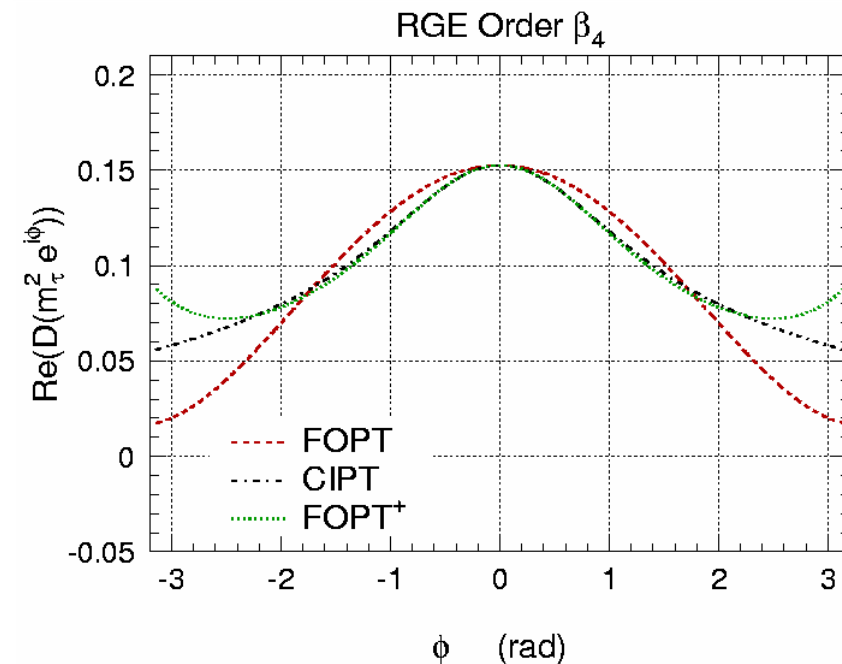
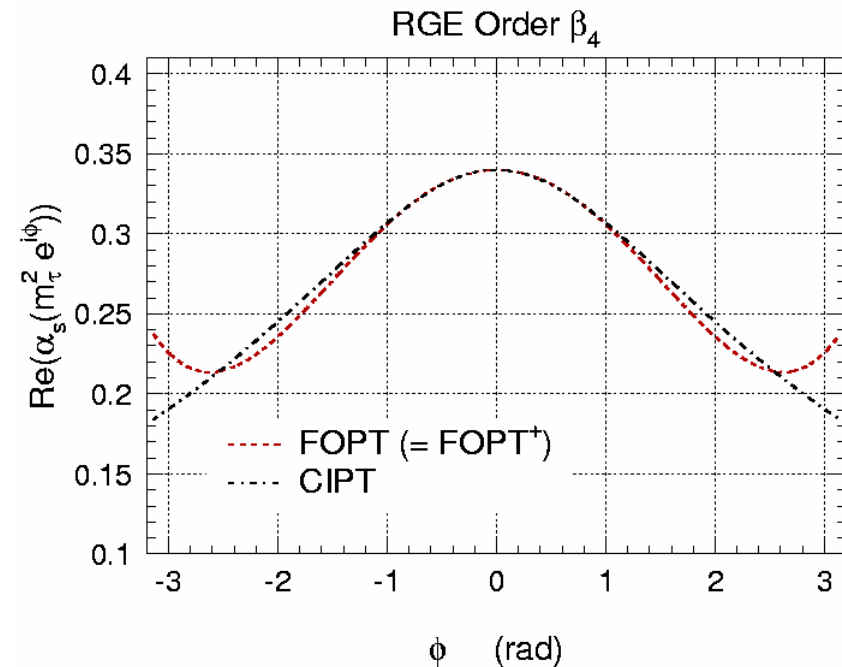
Tests of Integration Methods

$$\alpha_s \rightarrow D \rightarrow \delta^{(0)}$$

$$1 + \delta^{(0)} = -2\pi i \oint_{|s|=s_0} \frac{ds}{s_0} \left(1 - 2\frac{s}{s_0} + 2\left(\frac{s}{s_0}\right)^3 - \left(\frac{s}{s_0}\right)^4 \right) \cdot D(s)$$

$$\text{where: } D(s) = \frac{1}{4\pi^2} \sum_{n=0}^6 \tilde{K}_n(\xi) a_s^n(-\xi s)$$

- FOPT+: the same Taylor expansion for $\alpha_s(s)$ as FOPT, with no cut of the integration result

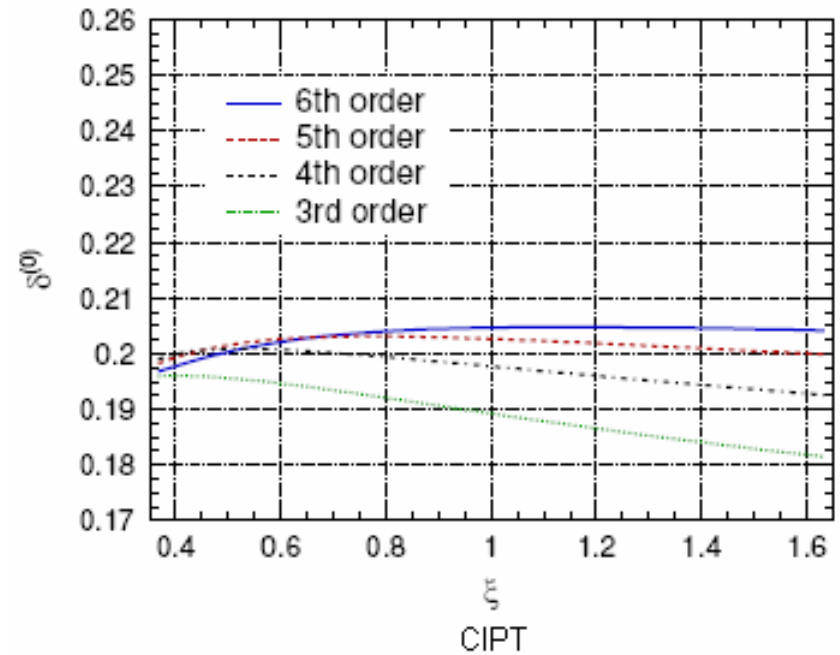
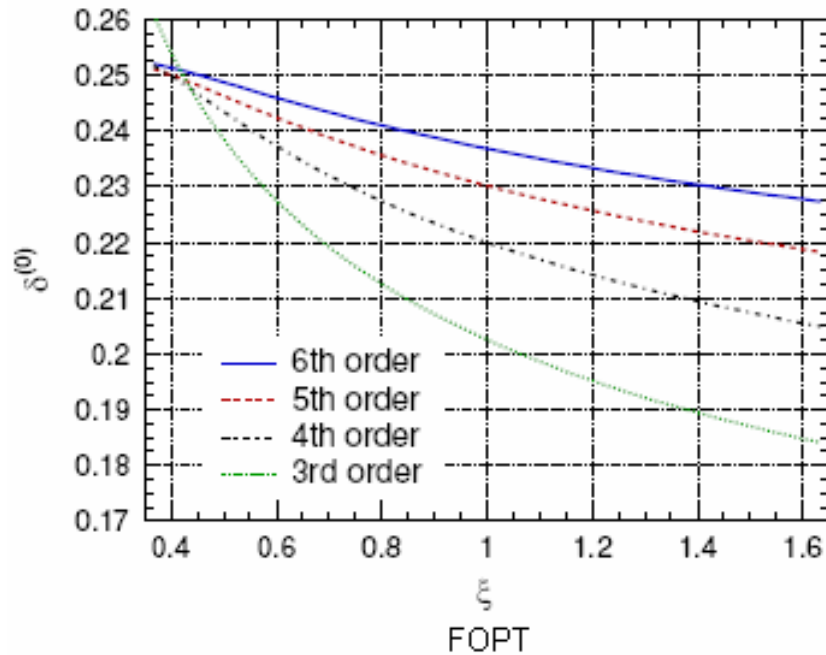


Numerical Tests of Integration Methods: $\delta^{(0)}$

Massless perturbative contribution $\delta^{(0)}$ computed for $\alpha_s(m_\tau^2) = 0.34$ with $K_{5,6}$ and β_4 estimated by assuming geometric growth. Higher unknown coefficients were set to zero.

Pert. Method	$n = 1$	$n = 2$	$n = 3$	$n = 4$	$(n = 5)$	$(n = 6)$	$\sum_{n=1}^4$	$\sum_{n=1}^5$	$\sum_{n=1}^6$
FOPT ($\xi = 1$)	0.1082	0.0609	0.0334	0.0174	0.0101	0.0067	0.2200	0.2302	0.2369
$\delta(\beta_4 \pm 100\%)$	0	0	0	0	0	± 0.0006	0	0	± 0.0006
$\delta(K_5 \pm 100\%)$	0	0	0	0	± 0.0056	± 0.0108	0	± 0.0056	± 0.0164
$\delta(K_6 \pm 100\%)$	0	0	0	0	0	± 0.0047	0	0	± 0.0047
$\delta(\xi \pm 0.63)$	-	-	-	-	-	-	$+0.0317$ -0.0151	$+0.0209$ -0.0119	$+0.0152$ -0.0095
CIPT ($\xi = 1$)	0.1476	0.0295	0.0121	0.0085	0.0049	0.0020	0.1977	0.2027	0.2047
$\delta(\beta_4 \pm 100\%)$	∓ 0.0003	∓ 0.0001	∓ 0.0001	∓ 0.0001	∓ 0.0001	∓ 0.0001	∓ 0.0006	∓ 0.0007	∓ 0.0008
$\delta(K_5 \pm 100\%)$	0	0	0	0	± 0.0049	0	0	± 0.0049	± 0.0049
$\delta(K_6 \pm 100\%)$	0	0	0	0	0	± 0.0020	0	0	± 0.0020
$\delta(\xi \pm 0.63)$	-	-	-	-	-	-	$+0.0032$ -0.0051	$+0.0005$ -0.0044	$+0.0001$ -0.0079

Integration Methods: Scale dependence



Conclusions of our tests:

- FOPT neglects part of contributions to the perturbative series included in CIPT
- FOPT generally does not satisfy to the RGE and uses Taylor expansion in a region where it badly converges
- It is due to the properties of the kernel that we do not get higher differences between FOPT and CIPT
- CIPT avoids many problems and is to be preferred *

*See however a recent study of the role of higher order contributions (with different approach and conclusions): M.Beneke and M. Jamin (arXiv:0806.3156)

Impact of Quark-Hadron Duality Violation

Q-H Duality Violation: OPE only part of the non-perturbative contributions, non-perturbative oscillating terms missed...

Two models to simulate the contribution of duality violating terms (M.A.Shifman hep-ph/0009131):

- instantons;
- resonances.

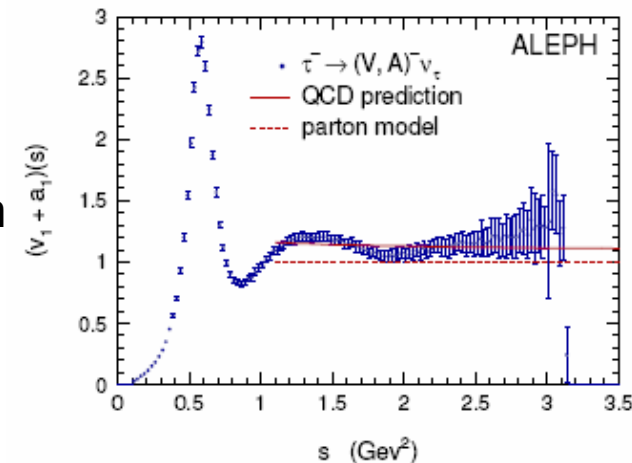
This contribution is added to the theoretical computation and the parameters of the models are chosen to match smoothly the V+A spectral function, near $s=m_\tau^2$.

Contributions to $\delta^{(0)}$ (~ 0.2):

- instantons: $< 4.5 \cdot 10^{-3}$
- resonances: $< 7 \cdot 10^{-4}$

Those contributions are within our systematic uncertainties.

This problem has also been considered by O. Cata, et al. arxiv: 0803.0246



Spectral Moments

- Exploit shape of spectral functions to obtain additional experimental information:

$$R_{\tau,U}^{k\ell}(s_0) = \int_0^{s_0} ds \left(1 - \frac{s}{s_0}\right)^k \left(\frac{s}{s_0}\right)^\ell \frac{dR_{\tau,U}(s_0)}{ds}$$

Le Diberder-Pich,
PL B289, 165 (1992)

The region where OPE fails and we have small statistics is suppressed.

- Theory prediction very similar to R_τ :

$$R_{\tau,U}^{k\ell}(s_0) \propto |V_{\text{CKM}}|^2 S_{\text{EW}} \left(P^{(k\ell)} + \delta^{(0,k\ell)} + \delta_{\text{EW}}^{(k\ell)} + \delta_U^{(2,m_q,k\ell)} + \sum_{D=4,6,\dots} \delta_U^{(D,k\ell)} \right)$$

with corresponding perturbative and nonperturbative OPE terms

- Because of the strong correlations, only four moments are used.
- We fit simultaneously $\alpha_s(m_\tau^2)$ and the leading $D=4,6,8$ nonperturbative contributions

Aleph Fit Results

- The combined fit of R_τ and spectral moments ($k=1, \ell=0,1,2,3$) gives (at $s_0=m_\tau^2$):

Parameter	Vector (V)	Axial-Vector (A)	V + A
$\alpha_s(m_\tau^2)$	$0.3474 \pm 0.0074^{+0.0063}_{-0.0074}$	$0.3345 \pm 0.0078^{+0.0063}_{-0.0074}$	$0.3440 \pm 0.0046^{+0.0063}_{-0.0074}$
$\delta^{(0)}$	0.2093 ± 0.0080	0.1988 ± 0.0087	0.2066 ± 0.0070
$\delta^{(2)}$	$(-3.2 \pm 3.0) \cdot 10^{-4}$	$(-5.1 \pm 3.0) \cdot 10^{-4}$	$(-4.3 \pm 2.0) \cdot 10^{-4}$
$\langle a_s GG \rangle$ (GeV ⁴)	$(-0.8 \pm 0.4) \cdot 10^{-2}$	$(-2.2 \pm 0.4) \cdot 10^{-2}$	$(-1.5 \pm 0.3) \cdot 10^{-2}$
$\delta^{(4)}$	$(0.1 \pm 1.5) \cdot 10^{-4}$	$(-5.9 \pm 0.1) \cdot 10^{-3}$	$(-3.0 \pm 0.1) \cdot 10^{-3}$
$\delta^{(6)}$	$(2.68 \pm 0.20) \cdot 10^{-2}$	$(-3.46 \pm 0.21) \cdot 10^{-2}$	$(-3.7 \pm 1.7) \cdot 10^{-3}$
$\delta^{(8)}$	$(-8.0 \pm 0.5) \cdot 10^{-3}$	$(0.5 \pm 0.5) \cdot 10^{-3}$	$(8.1 \pm 3.6) \cdot 10^{-4}$
Total δ_{NP}	$(1.89 \pm 0.25) \cdot 10^{-2}$	$(-3.11 \pm 0.16) \cdot 10^{-2}$	$(-5.9 \pm 1.4) \cdot 10^{-3}$
χ^2/DF	0.07	3.57	0.90

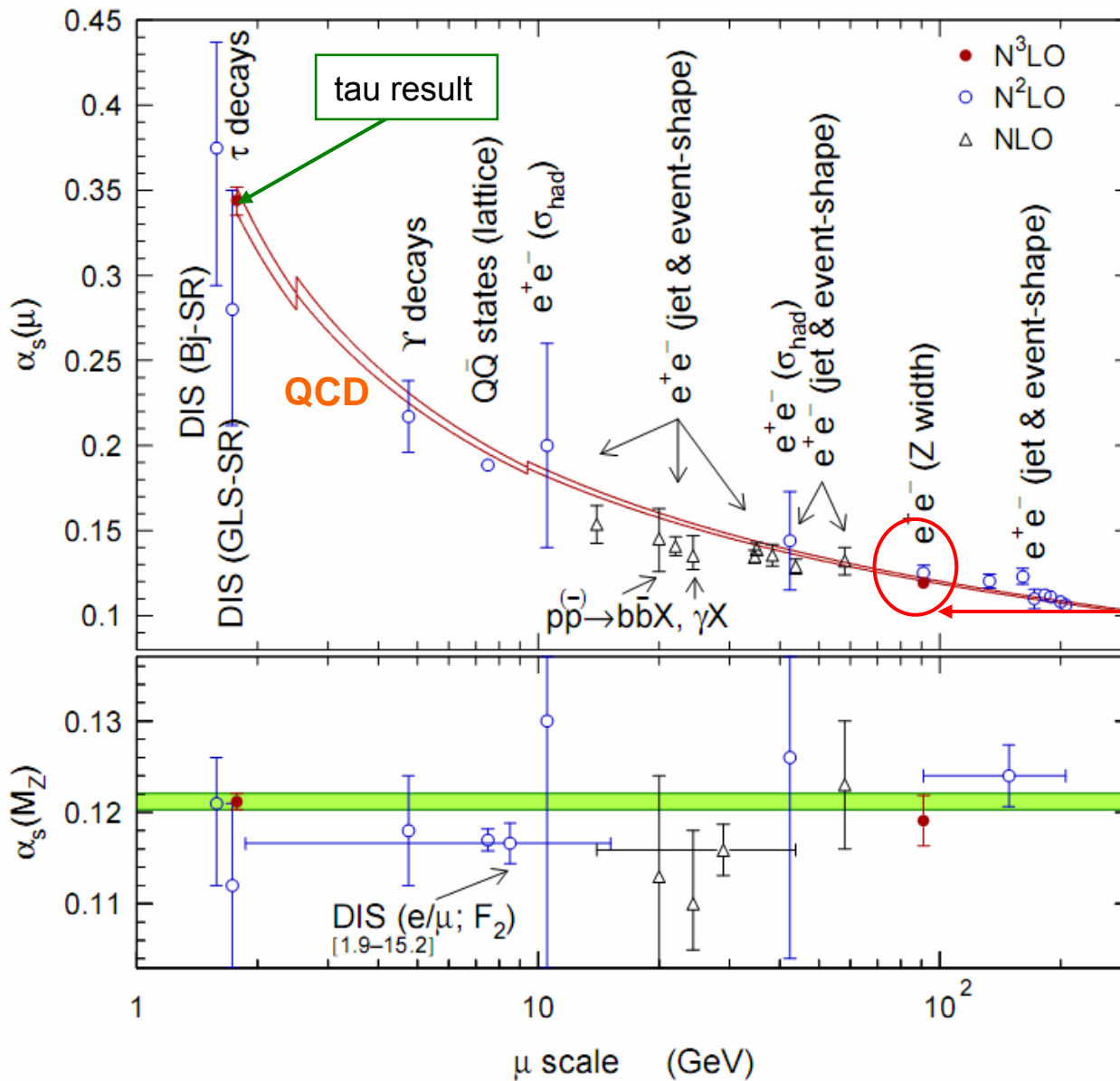
- Theory framework: tests \Rightarrow CIPT method preferred, no CIPT-vs.-FOPT syst.
- The fit to the V+A data yields:

$$\alpha_s(m_\tau^2) = 0.344 \pm 0.005_{\text{exp}} \pm 0.007_{\text{theo}}$$

- Using 4-loop QCD β -function and 3-loop quark-flavour matching yields:

$$\alpha_s(M_Z^2)_\tau = 0.1212 \pm 0.0005_{\text{exp}} \pm 0.0008_{\text{theo}} \pm 0.0005_{\text{evol}}$$

Overall comparison



Tau provides:

- among most precise $\alpha_s(M_Z^2)$ determinations;
- with $\alpha_s(M_Z^2)_Z$, the most precise test of asymptotic freedom (1.8-91GeV)

$$\alpha_s(m_Z^2)_Z = 0.1191 \pm 0.0027_{\text{fit}} \pm 0.0001_{\text{trunc}}$$

$$\alpha_s(M_Z^2)_\tau = 0.1212 \pm 0.0011$$

$$\alpha_s(M_Z^2)_\tau - \alpha_s(m_Z^2)_Z = 0.0021 \pm 0.0029$$

Z result

Conclusions

- Detailed studies of perturbative series: CIPT is to be preferred
- Contributions coming from duality violation are within systematic uncertainties
- $\alpha_s(m_\tau^2)$, extrapolated at M_Z scale, is among most precise values of $\alpha_s(M_Z^2)$
- $\alpha_s(m_\tau^2)$ and $\alpha_s(M_Z^2)$ from Z decays provide the most precise test of asymptotic freedom in QCD with a precision of 2.4%

backup

Tau Hadronic Spectral Functions

$$R_\tau = \frac{\Gamma(\tau^- \rightarrow \text{hadrons } \nu_\tau)}{\Gamma(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)} \approx N_C (|V_{ud}|^2 + |V_{us}|^2) \approx 3$$

neglecting QCD and EW corrections

$$d\Gamma(\tau^- \rightarrow \text{hadrons}^- \nu_\tau) = \frac{G_F^2}{4M_\tau} |V_{CKM}|^2 L_{\mu\nu} H^{\mu\nu} dPS$$

- Hadronic physics factorizes in (vector and axial-vector) Spectral Functions :

$$v_1/a_1[\tau^- \rightarrow V^- / A^- \nu_\tau] \propto \frac{\text{BR}[\tau^- \rightarrow V^- / A^- \nu_\tau]}{\text{BR}[\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau]} \frac{1}{N_{V/A}} \frac{dN_{V/A}}{ds} \frac{m_\tau^2}{(1-s/m_\tau^2)^2 (1+s/m_\tau^2)}$$

branching fractions
mass spectrum
kinematic factor

Fundamental ingredient relating long distance hadrons to short distance quarks (QCD)

• Optical Theorem: $\text{Im} \Pi_{\bar{u}d(s), V/A}^{(1,0)} = \frac{1}{2\pi} v_1/a_{1,0}(s)$

Currents Separation

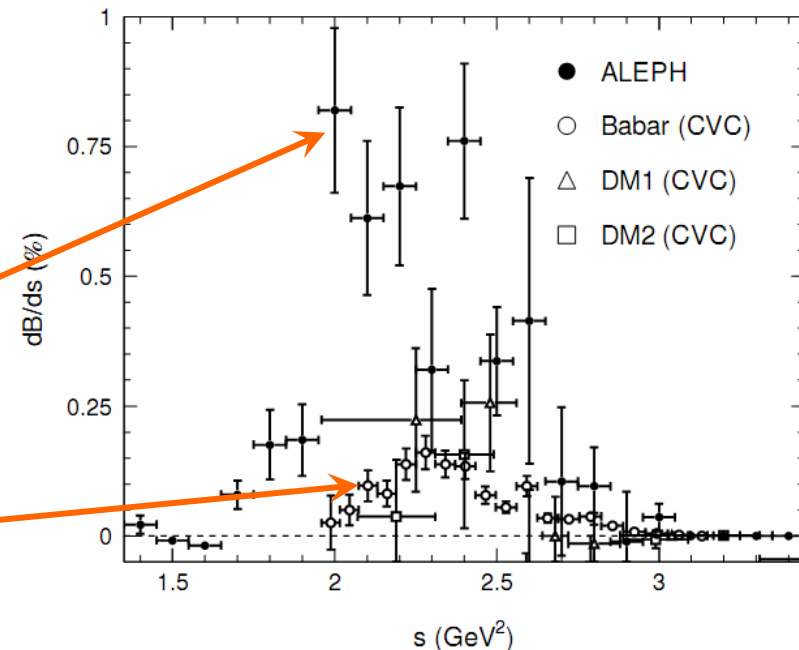
$$R_\tau = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$

Separation of V and A components:

- Straightforward for final states with only pions (using G-parity) :
 - even number of pions ($G = 1$): vector state
 - odd number of pions ($G = -1$): axial-vector state
- $K\bar{K}$ modes are generally not eigenstates of G-parity :
 - $K^- K^0$ is pure vector
 - $K\bar{K}\pi$ BABAR: $f_A = 0.833 \pm 0.024$
 - $K\bar{K}\pi\pi$ rarer modes: $f_A = 0.5 \pm 0.5$

ALEPH(V+A)

BABAR+CVC (V)



Experimental Measurements

- From measured leptonic branching ratios:

$$R_\tau = \frac{1 - B_e - B_\mu}{B_e} = \frac{1}{B_e^{uni}} - 1.9726 = 3.640 \pm 0.010$$

$g_e = g_\mu = g_\tau$

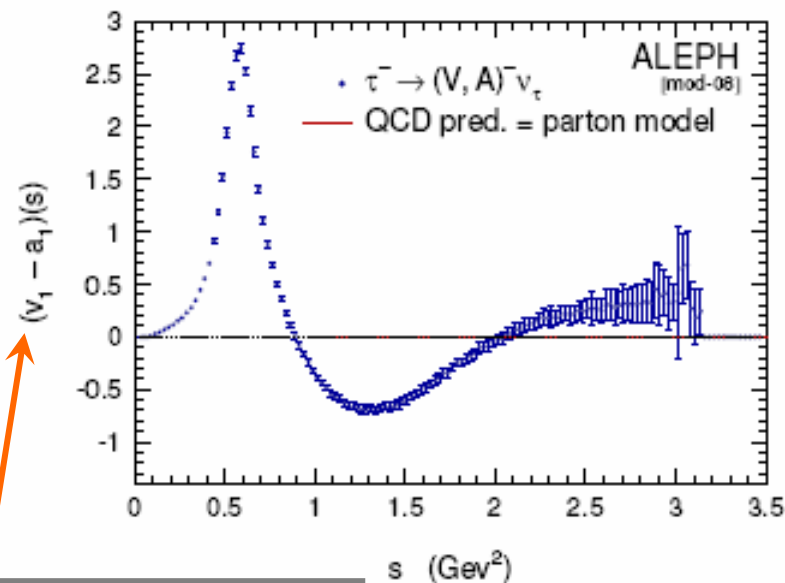
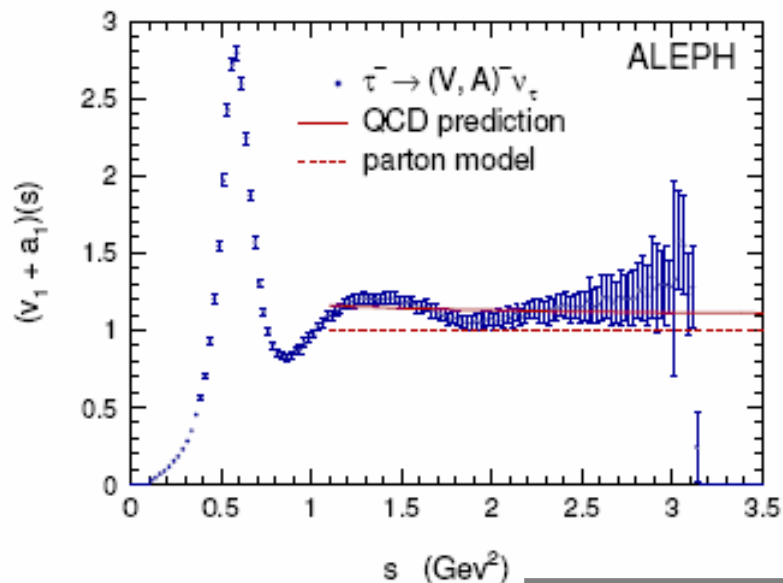
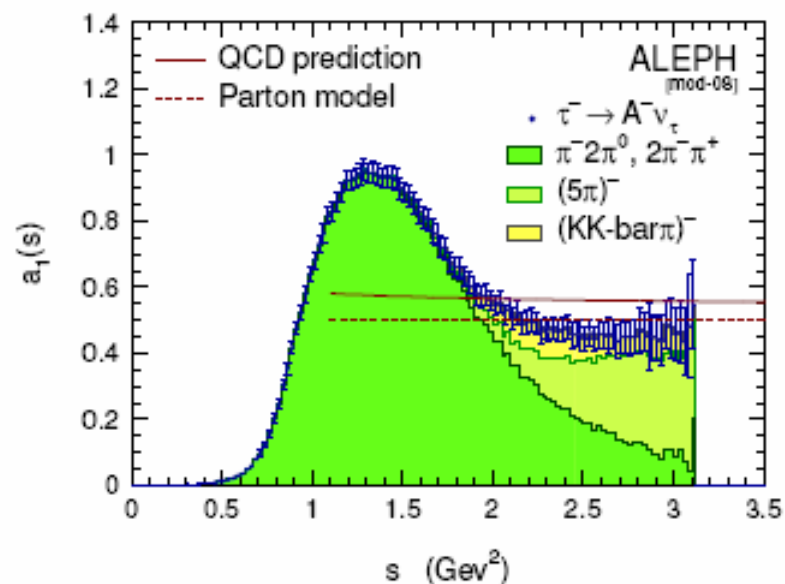
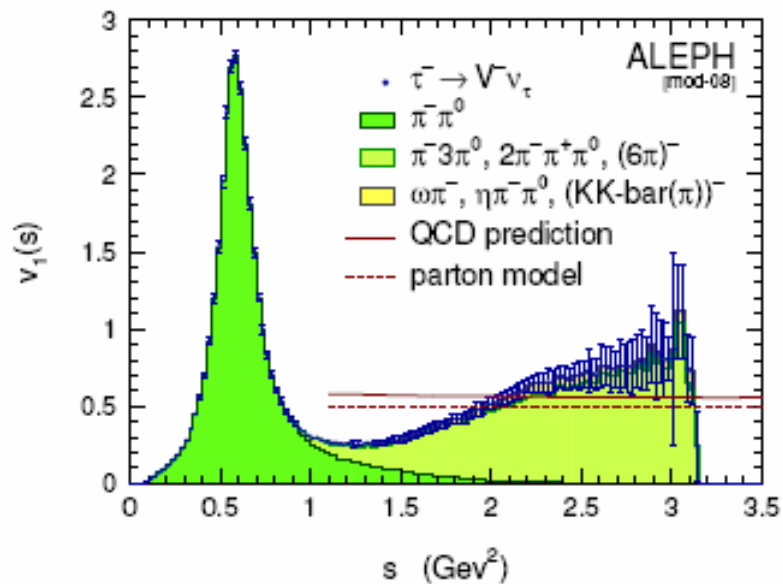
- Vector, Axial-Vector ($\Delta S = 0$) and Strange contributions ($\Delta S = 1$):

$$R_\tau = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$

$$R_{\tau,V} = 1.783 \pm 0.011_{\text{exp}} \pm 0.002_{V/A}$$

$$R_{\tau,A} = 1.695 \pm 0.011_{\text{exp}} \pm 0.002_{V/A}$$

$$R_{\tau,S} = 0.1615 \pm 0.0040 \quad (\text{incl. new results from BABAR+Belle})$$



Of purely nonperturbative origin

Fit details

- Although $\delta^{(0)}$ is the main contribution, and the one that provides the sensitivity to α_s , we must not forget the other terms in the OPE (i.e. Quark-Mass and Nonperturbative Contributions):
 - $D=2$ (mass dimension): quark-mass terms are $\propto m_q^2/s_0$, which is negligible for $q=u,d$
 - $D=4$: dominant contributions from gluon- and quark-field condensations (gluon condensate $\langle a_s GG \rangle$ is determined from data)
 - $D=6$: dominated by large number of four-quark dynamical operators that – assuming factorization (vacuum saturation) – can be reduced to an effective scale-independent operator $\rho a_s \langle qq\text{-bar} \rangle^2$ that is determined from data
 - $D=8$: structure of quark-quark, quark-gluon and four-gluon condensates absorbed in single phenomenological operator $\langle O_8 \rangle$ determined from data
- For practical reasons it is convenient to normalize the spectral moments:

$$D_{\tau,U}^{kl}(s_0) = \frac{R_{\tau,U}^{kl}(s_0)}{R_{\tau,U}(s_0)}$$